
Exercises to Relativistic Quantum Field Theory — Sheet 6

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Exercise 6.1 *Momentum of the quantised free scalar field (cont'd)* (1 point)

a) How does P^μ from Exercise 5.1 act on the one-particle wave function $\varphi_{\vec{p}}(t, \vec{x}) = \langle 0 | \phi(t, \vec{x}) | \vec{p} \rangle$ with fixed momentum \vec{p} ? The action of an operator A on $\varphi_{\vec{p}}(t, \vec{x})$ is defined by $A\varphi_{\vec{p}}(t, \vec{x}) = \langle 0 | \phi(t, \vec{x}) A | \vec{p} \rangle$.

b) Express the one-particle wave function $\phi_f(t, \vec{x})$ corresponding to the state

$$|f\rangle = \int d\vec{p} f(\vec{P}) | \vec{p} \rangle$$

in terms of $\phi_{\vec{p}}(t, \vec{x})$. Show that $\phi_f(t, \vec{x})$ satisfies the Klein-Gordon equation.

Exercise 6.2 *Identities of the scalar field operator* (1 point)

Consider the field operator $\phi(x)$ of the free, real Klein-Gordon field.

a) Show that

$$[\phi(x), \phi(y)] = \int d\vec{k} \left(e^{-ik(x-y)} - e^{+ik(x-y)} \right)$$

and argue why $[\phi(x), \phi(y)] = 0$ for $(x - y)^2 < 0$, as demanded by causality.

b) Prove the relation between time ordering and normal ordering:

$$\begin{aligned} T[\phi(x)\phi(y)] &\equiv \theta(x_0 - y_0)\phi(x)\phi(y) + \theta(y_0 - x_0)\phi(y)\phi(x) \\ &= : \phi(x)\phi(y) : + \langle 0 | T[\phi(x)\phi(y)] | 0 \rangle. \end{aligned}$$

Exercise 6.3 *Normalisation of multi-particle states* (0.5 points)

Show that the n -particle states in (bosonic) Fock space,

$$| \vec{p}_1, \dots, \vec{p}_n \rangle = a^\dagger(\vec{p}_1) a^\dagger(\vec{p}_2) \dots a^\dagger(\vec{p}_n) | 0 \rangle,$$

are normalised according to

$$\langle \vec{p}_1, \dots, \vec{p}_n | \vec{k}_1, \dots, \vec{k}_m \rangle = \delta_{mn} (2\pi)^{3n} \sum_{\pi \in S_n} \prod_i (2p_i^0) \delta^3(\vec{p}_i - \vec{k}_{\pi(i)}),$$

where the sum runs over all permutations S_n of the indices $\{1, \dots, n\}$.