Exercises to Group Theory for Physicists — Sheet 9

Prof. S. Dittmaier and Dr. P. Maierhöfer, Universität Freiburg, SS19

Exercise 9.1 *Isospin symmetry of hadrons* (2 points)

a) The ^{3}H nucleus (a *pnn* bound state) and the ^{3}He nucleus (a *ppn* bound state) form an isospin doublet ($I = 1/2$). Use isospin symmetry of strong interactions to predict the ratio of the cross sections for $p + d \to {}^3\mathrm{H} + \pi^+$ and $p + d \to {}^3\mathrm{He} + \pi^0$.

b) The baryon resonances Δ^{++}, Δ^+, Δ^0, and Δ^-, form an isospin quartet ($I = 3/2$) and can be produced via strong interactions in pion–nucleon collisions. Assuming isospin symmetry, what is the ratio of the production cross sections for $\pi^+ + p \to \Delta^{++}$ and $\pi^- + p \to \Delta^0$?

Exercise 9.2 *Euclidean group in two dimensions* (6 points)

The defining representation of the Euclidean group in two dimensions is given by the matrices

$$D(\vec{\theta}) = D(\vec{a}, \phi) = \begin{pmatrix} \cos\phi & -\sin\phi & a_1 \\ \sin\phi & \cos\phi & a_2 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \vec{a} \in \mathbb{R}^2, \quad \phi \in [0, 2\pi). \tag{1}$$

In the following, the group parameters sometimes will be generically labelled by $\vec{\theta} = (\theta_1, \theta_2, \theta_3) = (a_1, a_2, \phi)$.

a) Is the group compact, connected, simply connected?

b) Is the group abelian, simple, semisimple?

c) Give the functions f_A explicitly that encode group multiplication via $\theta''_A = f_A(\vec{\theta}\,', \vec{\theta})$, where $D(\vec{\theta}\,'') = D(\vec{\theta}\,')D(\vec{\theta})$. Derive the group generators $\mathcal{X}^A(\vec{\theta})$ at an arbitrary group element $D(\vec{\theta})$ from the functions f_A.

d) Calculate all structure constants $f^{AB}{}_C$ from the commutators $\left[\mathcal{X}^A(\vec{\theta}), \mathcal{X}^B(\vec{\theta})\right] = \mathrm{i} f^{AB}{}_C\, \mathcal{X}^C(\vec{\theta})$, in order to check that they do not depend on $\vec{\theta}$.

e) The matrices define the following Lie group transformations for the coordinates x_1, x_2,

$$\begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} \to \begin{pmatrix} x'_1 \\ x'_2 \\ 1 \end{pmatrix} = D(\vec{\theta}) \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}. \tag{2}$$

Derive the group generators $X^A(\vec{x})$ for these transformations from the functions F_a defined by $x'_a = F_a(\vec{\theta}, \vec{x})$ for finite $\vec{\theta}$.

f) Verify that the $X^A(\vec{x})$ obey the Lie algebra commutation relations. What is the physical meaning of the individual operators $X^A(\vec{x})$?