Exercises on Supersymmetry **Sheet 7**

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Exercise 15 *Chiral superfield in superspace* (6 points)

A chiral superfield $\Phi(x,\theta,\bar\theta)$ has the following decomposition in superspace,

$$\begin{aligned}\Phi(x,\theta,\bar\theta) &= \phi(x) - i(\theta\bar\sigma^\mu\bar\theta)\partial_\mu\phi(x) - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial^2\phi(x) \\ &\quad + \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}(\theta\theta)(\bar\theta\sigma^\mu\partial_\mu\psi(x)) - (\theta\theta)\mathcal{F}(x),\end{aligned} \tag{1}$$

with component fields $\phi(x)$, $\psi_a(x)$, and $\mathcal{F}(x)$.

a) Verify $\bar{\mathcal{D}}^{\dot a}\Phi(x,\theta,\bar\theta) = 0$, with covariant derivative $\bar{\mathcal{D}}^{\dot a} = i\left(\frac{\partial}{\partial\bar\theta_{\dot a}} - i\sigma^{\mu,\dot a b}\theta_b\partial_\mu\right)$.

b) Calculate the SUSY variations $\delta f = -i(\alpha\mathcal{Q} + \bar\alpha\bar{\mathcal{Q}})f$ for the component fields $f = \phi,\psi,\mathcal{F}$ with

$$\mathcal{Q}_a = i\left(\frac{\partial}{\partial\theta^a} + i\bar\sigma^\mu_{a\dot b}\bar\theta^{\dot b}\partial_\mu\right), \tag{2}$$

$$\bar{\mathcal{Q}}^{\dot a} = i\left(\frac{\partial}{\partial\bar\theta_{\dot a}} + i\sigma^{\mu,\dot a b}\theta_b\partial_\mu\right). \tag{3}$$

c) Show that the product of two chiral superfields is again a chiral superfield. Determine the component fields of $\Phi'' = \Phi\Phi'$ in terms of the component fields of the chiral superfields Φ and Φ'.

Please turn over!

Exercise 16 *Integration in superspace* (4 points)

Integration over Grassmann-valued coordinates θ_a and $\bar\theta^{\dot a}$ in superspace can be defined (together with linearity of the integral) in terms of the elementary integrals

$$\int d\theta_a\, 1 = 0, \qquad \int d\theta_a\, \theta^b = \delta_a^b, \qquad \int d^2\theta = \int d\theta_2 \int d\theta_1 = \frac{1}{2}\int d\theta^a \int d\theta_a, \tag{4}$$

$$\int d\bar\theta^{\dot a}\, 1 = 0, \qquad \int d\bar\theta^{\dot a}\, \bar\theta_{\dot b} = \delta_{\dot b}^{\dot a}, \qquad \int d^2\bar\theta = \int d\bar\theta^{\dot 1} \int d\bar\theta^{\dot 2} = \frac{1}{2}\int d\bar\theta_{\dot a} \int d\bar\theta^{\dot a}, \tag{5}$$

$$\int d^4\theta = \int d^2\theta \int d^2\bar\theta. \tag{6}$$

a) Calculate $\int d^2\theta\, (\theta\theta)$, $\int d^2\bar\theta\, (\bar\theta\bar\theta)$ and $\int d^4\theta\, (\theta\theta)(\bar\theta\bar\theta)$.

b) Show that the functions $\delta^{(2)}(\theta) = -\frac{1}{2}(\theta\theta)$ and $\delta^{(2)}(\bar\theta) = -\frac{1}{2}(\bar\theta\bar\theta)$ act like "δ-functions" within the superspace integral, by integrating them together with an arbitrary superfield $\mathcal{S}(x, \theta, \bar\theta)$.

c) For an arbitrary superfield $\mathcal{S}(x, \theta, \bar\theta)$ and a chiral superfield $\Phi(x, \theta, \bar\theta)$ show

$$-\frac{1}{2}\int d^4x \int d^4\theta\, \mathcal{S}(x, \theta, \bar\theta) = \int d^4x\, [\mathcal{S}]_D\,, \tag{7}$$

$$\frac{1}{2}\int d^4x \int d^4\theta\, \delta^{(2)}(\bar\theta)\, \Phi(x, \theta, \bar\theta) = \int d^4x\, [\Phi]_{\mathcal{F}}\,, \tag{8}$$

where $[\ldots]_D$ and $[\ldots]_{\mathcal{F}}$ denote the "D"- and "$\mathcal{F}$" terms of the superfields.

d) Give the action $S = \int d^4x\, \mathcal{L}$ of the Wess–Zumino-model in terms of the integral $\int d^4x \int d^4\theta\, [\ldots]$.