Exercises on Supersymmetry Sheet 6

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Exercise 13 *Wess–Zumino model* (8 points)

After elimination of the auxiliary fields the Lagrangian density is given as

$$\mathcal{L} = \mathcal{L}_{\text{bos}} + \mathcal{L}_{\text{ferm}} + \mathcal{L}_{\text{Yuk}}, \tag{1}$$

$$\mathcal{L}_{\text{bos}} = (\partial^\mu\phi)^\dagger(\partial_\mu\phi) - m^2\phi^\dagger\phi + \sqrt{2}gm(\phi^\dagger\phi)(\phi+\phi^\dagger) - 2g^2(\phi^\dagger\phi)^2, \tag{2}$$

$$\mathcal{L}_{\text{ferm}} = i\psi^a\bar{\sigma}^\mu_{a\dot{b}}\partial_\mu\bar{\psi}^{\dot{b}} - \frac{m}{2}(\psi^a\psi_a + \bar{\psi}_{\dot{a}}\bar{\psi}^{\dot{a}}), \tag{3}$$

$$\mathcal{L}_{\text{Yuk}} = \sqrt{2}g(\phi\psi^a\psi_a + \phi^\dagger\bar{\psi}_{\dot{a}}\bar{\psi}^{\dot{a}}), \tag{4}$$

where ϕ denotes a complex scalar field and ψ^a a (Grassmann-valued) Weyl spinor field.

a) Derive the equations of motion for ϕ, $\phi^\dagger$, ψ, and $\bar{\psi}$.

b) Calculate the SUSY variation $[Q_a, \mathcal{L}]$, where

$$[Q_a, \phi] = -i\sqrt{2}\psi_a, \qquad [Q_a, \phi^\dagger] = 0, \tag{5}$$

$$\{Q_a, \psi_b\} = i\sqrt{2}\epsilon_{ab}\phi^\dagger(m - \sqrt{2}g\phi^\dagger), \qquad \{Q_a, \bar{\psi}_{\dot{b}}\} = \sqrt{2}\bar{\sigma}^\mu_{a\dot{b}}\partial_\mu\phi^\dagger, \tag{6}$$

and show that $[Q_a, \mathcal{L}] \sim 0$, i.e. the commutator vanishes up to surface terms and terms that vanish because of the equations of motion.

(intermediate results: $[Q_a, \mathcal{L}_{\text{bos}}] \sim 0$, $[Q_a, \mathcal{L}_{\text{ferm}}] \sim -[Q_a, \mathcal{L}_{\text{Yuk}}] \sim -2ig\phi^\dagger\psi_a[m(\phi^\dagger + 2\phi) - 4\sqrt{2}g\phi^\dagger\phi]$.)

Exercise 14 *Covariant derivatives in superspace* (4 points)

The covariant derivatives in superspace are defined by

$$\mathcal{D}_a = i\left(\frac{\partial}{\partial\theta^a} - i\bar{\sigma}^\mu_{a\dot{b}}\bar{\theta}^{\dot{b}}\partial_\mu\right), \qquad \bar{\mathcal{D}}^{\dot{a}} = i\left(\frac{\partial}{\partial\bar{\theta}_{\dot{a}}} - i\sigma^{\mu,\dot{a}b}\theta_b\partial_\mu\right). \tag{7}$$

a) Verify the fundamental anticommutators

$$\{\mathcal{D}_a, \mathcal{D}_b\} = 0, \qquad \{\bar{\mathcal{D}}_{\dot{a}}, \bar{\mathcal{D}}_{\dot{b}}\} = 0, \qquad \{\mathcal{D}_a, \bar{\mathcal{D}}_{\dot{b}}\} = -2i\bar{\sigma}^\mu_{a\dot{b}}\partial_\mu. \tag{8}$$

b) Show that

$$[\bar{\mathcal{D}}_{\dot{b}}\bar{\mathcal{D}}^{\dot{b}}, \mathcal{D}_a] = 4i\bar{\sigma}^\mu_{a\dot{b}}\partial_\mu\bar{\mathcal{D}}^{\dot{b}}, \qquad [\mathcal{D}^b\mathcal{D}_b, \bar{\mathcal{D}}^{\dot{a}}] = 4i\sigma^{\mu,\dot{a}b}\partial_\mu\mathcal{D}_b. \tag{9}$$