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Lecture "Supersymmetry"

1 Conventions for Weyl spinors

- Weyl spinors (anticommuting!): covariant: ξ_a, contravariant: $\bar{\xi}^{\dot{a}}$.
- Spinor metric:

$$\epsilon = (\epsilon^{ab}) = (\epsilon_{ab}) = (\epsilon^{\dot{a}\dot{b}}) = (\epsilon_{\dot{a}\dot{b}}) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{1}$$

- Raising / lowering of indices:

$$\xi^a = \epsilon^{ab}\xi_b, \quad \xi_a = \xi^b\epsilon_{ba}, \quad \bar{\xi}^{\dot{a}} = \epsilon^{\dot{a}\dot{b}}\bar{\xi}_{\dot{b}}, \quad \bar{\xi}_{\dot{a}} = \bar{\xi}^{\dot{b}}\epsilon_{\dot{b}\dot{a}}. \tag{2}$$

- Complex conjugation:

$$\bar{\xi}_{\dot{a}} = (\xi_a)^*, \qquad \xi^a = (\bar{\xi}^{\dot{a}})^*, \qquad (\xi_a\eta_b)^* = \bar{\eta}_{\dot{b}}\bar{\xi}_{\dot{a}}, \quad \text{etc.} \tag{3}$$

- Spinor product:

$$\xi\eta \equiv \xi^a\eta_a = \epsilon^{ab}\xi_b\eta_a = \eta\xi, \tag{4}$$

$$\bar{\xi}\bar{\eta} \equiv \bar{\xi}_{\dot{a}}\bar{\eta}^{\dot{a}} = \epsilon_{\dot{a}\dot{b}}\bar{\xi}^{\dot{a}}\bar{\eta}^{\dot{b}} = \bar{\eta}\bar{\xi} = (\xi\eta)^*. \tag{5}$$

- Schouten identity and spinor relations:

$$0 = \epsilon^{ab}\epsilon^{cd} + \epsilon^{ac}\epsilon^{db} + \epsilon^{ad}\epsilon^{bc}, \quad \xi^a\eta^b = \xi^b\eta^a - \epsilon^{ab}(\xi\eta), \quad \xi^a\xi^b = -\frac{1}{2}\epsilon^{ab}(\xi\xi), \tag{6}$$

$$0 = \epsilon^{\dot{a}\dot{b}}\epsilon^{\dot{c}\dot{d}} + \epsilon^{\dot{a}\dot{c}}\epsilon^{\dot{d}\dot{b}} + \epsilon^{\dot{a}\dot{d}}\epsilon^{\dot{b}\dot{c}}, \quad \bar{\xi}^{\dot{a}}\bar{\eta}^{\dot{b}} = \bar{\xi}^{\dot{b}}\bar{\eta}^{\dot{a}} + \epsilon^{\dot{a}\dot{b}}(\bar{\xi}\bar{\eta}), \quad \bar{\xi}^{\dot{a}}\bar{\xi}^{\dot{b}} = +\frac{1}{2}\epsilon^{\dot{a}\dot{b}}(\bar{\xi}\bar{\xi}), \tag{7}$$

- Differentiation wrt Weyl spinors:

$$\frac{\partial}{\partial\theta^a}\theta^b = \delta_a^b, \quad \frac{\partial}{\partial\theta_a} = \epsilon^{ab}\frac{\partial}{\partial\theta^b}, \quad \frac{\partial}{\partial\theta_a}\theta_b = -\delta_b^a, \tag{8}$$

$$\frac{\partial}{\partial\bar{\theta}_{\dot{a}}}\bar{\theta}_{\dot{b}} = \delta_{\dot{b}}^{\dot{a}}, \quad \frac{\partial}{\partial\bar{\theta}^{\dot{a}}} = \frac{\partial}{\partial\bar{\theta}_{\dot{b}}}\epsilon_{\dot{b}\dot{a}}, \quad \frac{\partial}{\partial\bar{\theta}^{\dot{a}}}\bar{\theta}^{\dot{b}} = -\delta_{\dot{a}}^{\dot{b}}, \tag{9}$$

2 Weyl spinors and 4-vectors

- Minkowski metric:

$$(g^{\mu\nu}) = \mathrm{diag}(1, -1, -1, -1). \tag{10}$$

- Clebsch–Gordan coefficients:

$$\sigma^\mu = (\sigma^{\mu,\dot{a}b}) = (\mathbf{1}, \boldsymbol{\sigma}), \qquad \bar{\sigma}^\mu = (\sigma^\mu_{a\dot{b}}) = (\mathbf{1}, -\boldsymbol{\sigma}). \tag{11}$$

- Properties of σ matrices:

$$\sigma^\mu_{\dot{a}b} = \sigma^{\mu,\dot{c}d}\epsilon_{\dot{c}\dot{a}}\epsilon_{db}, \qquad \bar{\sigma}^{\mu,a\dot{b}} = \epsilon^{ac}\epsilon^{\dot{b}\dot{d}}\bar{\sigma}^\mu_{c\dot{d}}, \tag{12}$$

$$\bar{\sigma}^\mu_{a\dot{b}} = (\sigma^\mu_{\dot{a}b})^*, \qquad \sigma^\mu_{\dot{a}b} = \bar{\sigma}^\mu_{b\dot{a}}, \tag{13}$$

$$\sigma^\mu_{\dot{a}b}\sigma^{\nu,\dot{a}b} = 2g^{\mu\nu}, \qquad \sigma^\mu_{\dot{a}b}\sigma^{\nu,\dot{a}c} + \sigma^\nu_{\dot{a}b}\sigma^{\mu,\dot{a}c} = 2g^{\mu\nu}\delta^c_b, \tag{14}$$

$$\sigma^\mu_{\dot{a}b}\sigma_{\mu,\dot{c}d} = 2\epsilon_{\dot{a}\dot{c}}\epsilon_{bd}. \tag{15}$$

- 4-vectors from Weyl spinors:

$$\bar{\xi}\sigma^\mu\eta \equiv \bar{\xi}_{\dot{a}}\sigma^{\mu,\dot{a}b}\eta_b = \bar{\xi}^{\dot{a}}\sigma^\mu_{\dot{a}b}\eta^b, \tag{16}$$

$$\xi\bar{\sigma}^\mu\bar{\eta} \equiv \xi^a\bar{\sigma}^\mu_{a\dot{b}}\bar{\eta}^{\dot{b}} = \xi_a\bar{\sigma}^{\mu,a\dot{b}}\bar{\eta}_{\dot{b}} = -\bar{\eta}\sigma^\mu\xi. \tag{17}$$

- Relations:

$$\xi^a\bar{\eta}^{\dot{b}} = \tfrac{1}{2}(\xi\bar{\sigma}^\mu\bar{\eta})\bar{\sigma}^{a\dot{b}}_\mu, \tag{18}$$

$$(\xi\bar{\sigma}^\mu\bar{\eta})(\xi\bar{\sigma}^\nu\bar{\eta}) = \tfrac{1}{2}(\xi\xi)(\bar{\eta}\bar{\eta})g^{\mu\nu}. \tag{19}$$

3 Dirac- and Majorana spinors

- Dirac matrices in chiral representation:

$$\gamma^\mu = \begin{pmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{pmatrix} = \begin{pmatrix} 0 & (\bar{\sigma}^\mu_{a\dot{b}}) \\ (\sigma^{\mu,\dot{a}b}) & 0 \end{pmatrix}, \quad \gamma_5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{20}$$

- Dirac spinors:

$$\Psi = \begin{pmatrix} (\xi_a) \\ (\bar{\eta}^{\dot{a}}) \end{pmatrix}, \qquad \overline{\Psi} = \left((\eta^a), \quad (\bar{\xi}_{\dot{a}}) \right), \tag{21}$$

$$\overline{\Psi}\Psi = \xi\eta + \bar{\xi}\bar{\eta}, \qquad \overline{\Psi}\gamma^\mu\Psi = (\bar{\xi}\sigma^\mu\xi) + (\eta\bar{\sigma}^\mu\bar{\eta}), \tag{22}$$

$$\Psi_{\mathrm{c}} = C\overline{\Psi}^{\mathrm{T}} = C\gamma_0^{\mathrm{T}}\Psi^* = \begin{pmatrix} (\eta_a) \\ (\bar{\xi}^{\dot{a}}) \end{pmatrix}, \quad C = -\mathrm{i}\gamma^0\gamma^2, \quad C\gamma_0^{\mathrm{T}} = \begin{pmatrix} 0 & -\epsilon \\ \epsilon & 0 \end{pmatrix}. \tag{23}$$

- Majorana spinors:

$$\Psi = \begin{pmatrix} (\xi_a) \\ (\bar{\xi}^{\dot{a}}) \end{pmatrix} = \Psi_{\mathrm{c}}, \qquad \overline{\Psi} = \left((\xi^a), \quad (\bar{\xi}_{\dot{a}}) \right). \tag{24}$$

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