Exercises "Modern methods of Quantum Chromodynamics" **WS 14/15**

Problem 16 (4 Points) *Four-quark amplitudes*

Consider the quark-antiquark scattering of identical quark flavours:

$$q\bar{q} \to q\bar{q}$$

a) Show that the helicity amplitude $\mathcal{M}(q^L\bar{q}^R \to q^L\bar{q}^R)$ can be written as

$$\mathrm{i}\mathcal{M}(q^L_{i_1,p_1}\bar{q}^{R,i_2}_{p_2} \to q^L_{j_1,k_1}\bar{q}^{R,j_2}_{k_2}) = 2\mathrm{i}g_s^2\left[T^{a,j_2}_{j_1}T^{a,i_1}_{i_2}\frac{\langle k_1p_2\rangle^2}{\langle p_1p_2\rangle\langle k_2k_1\rangle} - T^{a,j_2}_{i_2}T^{a,i_1}_{j_1}\frac{\langle p_2k_1\rangle^2}{\langle p_1k_1\rangle\langle k_2p_2\rangle}\right]$$

b) Compute the helicity amplitudes $\mathcal{M}(q^L\bar{q}^R \to q^R\bar{q}^L)$ and $\mathcal{M}(q^L\bar{q}^L \to q^L\bar{q}^L)$

Problem 17 (4 Points) *Colour structures*

Consider the scattering amplitudes for processes with two quarks, two antiquarks and one gluon,

$$\mathcal{M}(q_i, \bar{q}^j, Q_k, \bar{Q}^l, g^a),$$

with two different quark flavours q and Q where all particles are treated as outgoing. The indices i, j, k, l denote the quark colour and the index a the colour of the gluon. Draw the contributing Feynman diagrams and show that the colour structures contributing to the amplitude can be chosen as

$$T^{a,l}_i\delta^j_k\,, \quad \delta^l_iT^{a,j}_k\,, \quad \frac{1}{N_c}T^{a,j}_i\delta^l_k\,, \quad \frac{1}{N_c}\delta^j_iT^{a,l}_k.$$

Problem 18 (2 Points) *Vanishing amplitudes*

Argue that scattering amplitudes for an arbitrary number of gluons vanish if all gluons or all gluon but one have the same helicity, i.e.

$$M(g_1^+, \ldots, \ldots g_n^+) = 0, \qquad M(g_1^+, \ldots, g_j^-, \ldots g_n^+) = 0,$$

and analogously for the amplitudes with opposite helicities. Here all gluons are taken as outgoing. You can argue using appropriate choices for the reference spinors of the gluon polarization vectors.