Exercise 6.1 (1 point) *Quark–antiquark scattering*

Consider the scattering process $q_i(p_1)\bar{q}_j(p_2) \to q_k(k_1)\bar{q}_l(k_2)$ in lowest order of perturbation theory in QCD, where the indices i, j, k, l denote the flavour quantum numbers of the quarks. The masses of the quarks as well as the electromagnetic couplings of the quarks to the photons can be neglected in this example.

a) Draw the contributing Feynman diagrams for the following cases:

- $i = j$, $i \neq k$;
- $i \neq j$, $i = k$;
- $i = j = k$.

b) Write down the transition matrix element $\mathcal{M}_s$ for the case $i = j$, $i \neq k$ (“s channel”). Show that it is independent of the gauge parameter ξ of the gluon propagator.

c) Calculate the spin- and colour-averaged squared matrix element

$$\overline{|\mathcal{M}_s|^2} = \frac{1}{4N_C^2} \sum_{pol.,colours} |\mathcal{M}_s|^2$$

and determine the differential cross section $d\sigma_s/d\cos\theta$ as well as the total cross section σ_s.
(Hint: You can make use of the result of Exercise 2.1.)

d) The squared matrix element $\overline{|\mathcal{M}_t|^2}$ for the case $i \neq j$, $i = k$ (“t channel”) can be obtained by identifying it with the s channel (via “crossing”) without need for a longer calculation. How do you have to normalize $\overline{|\mathcal{M}_t|^2}$? Write down $\overline{|\mathcal{M}_t|^2}$ explicitly.

Please turn over!

Exercise 6.2 (1 point) *3-jet production in e^-e^+ collisions*

Consider the process $e^-(p_-)e^+(p_+) \to \gamma^*(Q) \to q(k_1)\bar{q}(k_2)g(q)$ in the theory of "QCD$\otimes$QED" where the photonic interaction of the quarks is added to QCD. The momenta of the respective particles are indicated in brackets.

a) Draw the contributing Feynman diagrams in Born approximation and write down the matrix element in the form of

$$\mathcal{M}(e^-e^+ \to q\bar{q}g) = \mathcal{M}^\mu(e^-e^+ \to \gamma^*)\, G^{AA}_{\mu\nu}\, \mathcal{M}^\nu(\gamma^* \to q\bar{q}g)\ ,$$

where $G^{AA}_{\mu\nu}$ denotes the free propagator of the photon $\gamma(Q)$.

b) Show that the matrix element is independent of the gauge parameter ξ of the photon propagator. In order to do so, show that the identity

$$Q_\mu \mathcal{M}^\mu(\gamma^* \to q\bar{q}g) = 0$$

holds upon using the Dirac equation for the external spinors.

c) The complete matrix element of the process has the form

$$\mathcal{M}(e^-e^+ \to q\bar{q}g) = \varepsilon^\mu_a(q)^*\, T^a_\mu,$$

where $\varepsilon^\mu_a(q)^*$ denotes the polarisation vector of the outgoing gluon (with colour index a and momentum q). Show that

$$q^\mu\, T^a_\mu = 0.$$

Exercise 6.3 (1 point) *BRS transformation in covariant gauge*

After quantization, the Lagrangian of a Yang–Mills theory is given by $\mathcal{L} = \mathcal{L}_A + \mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{FP}}$ in covariant gauge with

$$\begin{aligned}
\mathcal{L}_A &= -\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu}, \qquad & F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gC^{abc}A^b_\mu A^c_\nu, \\
\mathcal{L}_{\text{fix}} &= -\frac{1}{2\xi}(\partial^\mu A^a_\mu)^2, \qquad & \mathcal{L}_{\text{FP}} &= -\,\bar{u}^a\partial^\mu D^{ab}_\mu u^b,
\end{aligned}$$

with the covariant derivative of the adjoint representation is $D^{ab}_\mu = \delta^{ab}\partial_\mu + gC^{abc}A^c_\mu$. Show the invariance $\delta_{\text{BRS}}\,[\mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{FP}}] = 0$ under the BRS variation

$$\begin{aligned}
\delta_{\text{BRS}}\, A^a_\mu(x) &= D^{ab}_\mu u^b(x)\delta\bar{\lambda}, \\
\delta_{\text{BRS}}\, \bar{u}^a(x) &= \frac{1}{\xi}\,\partial^\mu A^a_\mu(x)\,\delta\bar{\lambda}, \qquad \delta_{\text{BRS}}\, u^a(x) = -\frac{g}{2}C^{abc}u^b(x)u^c(x)\,\delta\bar{\lambda}
\end{aligned}$$

with infinitesimal global Grassmann variables $\delta\bar{\lambda}$. Start with showing that

$$\delta_{\text{BRS}}\,[D^{ab}_\mu u^b] = 0\,.$$