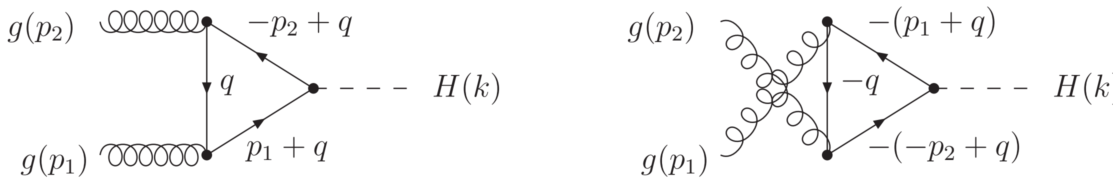
Exercise 12.1 (3 points) *Higgs–gluon vertex*

Via the Yukawa interaction discussed in Exercise 11.2, quark loops induce an effective Higgs–gluon coupling which, at one-loop level, is mediated by the following diagrams:



This process is the dominant production channel for the Higgs boson at the Tevatron and the LHC.

a) Show that, for on-shell gluons ($p_1^2 = p_2^2 = 0$), the vertex function of the Higgs–gluon vertex can be decomposed into the form

$$\Gamma_{\mu\nu}^{g_a g_b H}(p_1, p_2, k) = i\delta^{ab}\left[F_1(p_1, p_2, k)\left(g_{\mu\nu} - \frac{p_{2\mu}p_{1\nu}}{(p_1\cdot p_2)}\right) + F_2(p_1, p_2, k)\frac{p_{1\mu}p_{2\nu}}{(p_1\cdot p_2)}\right] .$$

Start with the general ansatz for $\Gamma_{\mu\nu}^{g_a g_b H}$ in form of the linear combination of all tensors of rank 2 which can be built with $g_{\mu\nu}$ and the momenta $p_{1,2}$. Apply then the gauge-invariance conditions $p_1^\mu\,\Gamma_{\mu\nu}^{ggH} = p_2^\nu\,\Gamma_{\mu\nu}^{ggH} = 0$.

b) Calculate the form factor $F_1(p_1, p_2, k)$ for $(p_1+p_2)^2 = M_H^2$, where M_H is the mass of the Higgs boson. Proceed with the following steps:

(1) By parameterizing the loop momentum as given in the diagrams, the numerators of both diagrams can be combined. Derive the following form for the sum of both diagrams:

$$\Gamma_{\mu\nu}^{g_a g_b H}(p_1, p_2, k) = \mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\frac{c_{ggH}\,\delta_{ab}\,\mathrm{Tr}[N_{\mu\nu}(p_1, p_2, k, m_q)]}{[q^2-m_q^2][(p_1+q)^2-m_q^2][(-p_2+q)^2-m_q^2]}.$$

The factor c_{ggH} comprises all coupling constants, colour factors, and powers of i, and m_q is the mass of the quark in the loop.

Please turn over!

(2) Project the tensor structure of $\Gamma^{g_a g_b H}_{\mu\nu}$ on the form factor F_1 with the help of the identity

$$P_{\mu\nu}P^{\mu\nu} = (D-2)\ , \quad P^{\mu\nu} \equiv g^{\mu\nu} - \frac{p_2^\mu p_1^\nu}{(p_1\cdot p_2)}.$$

The trace in the numerator results in

$$\mathrm{Tr}[N^{\mu\nu}P_{\mu\nu}] = 4m_q\left[2(D-1)m_q^2 - (D-2)M_H^2 - 2(D-5)q^2 - \frac{16(p_1\cdot q)(p_2\cdot q)}{M_H^2}\right]\ .$$

(3) Reduce the loop integral to scalar integrals. The limit of $D \to 4$ results in

$$F_1(p_1,p_2,k) = \frac{\alpha_s}{2\pi}\frac{m_q^2}{v}\left[(4m_q^2 - M_H^2)C_0(0,0,M_H^2,m_q^2,m_q^2,m_q^2) + 2\right]\ .$$

Outlook: The results above can be used to calculate the partial decay width of a Higgs boson into gluons. The matrix element $\mathcal{M}^{ab\lambda\lambda'}$ can be expressed using the form factor F_1 as

$$\mathcal{M}^{ab\lambda\lambda'} = i\delta^{ab}F_1(p_1,p_2,k)\left(g_{\mu\nu} - \frac{p_{2\mu}p_{1\nu}}{(p_1\cdot p_2)}\right)\varepsilon_\lambda^{\mu,*}(p_1)\varepsilon_{\lambda'}^{\nu,*}(p_2)\ ,$$

with ε_λ, $\varepsilon_{\lambda'}$ being the polarization vectors of the gluons and the on-shell conditions $p_{1\mu}\varepsilon_\lambda^{\mu,*}(p_1) = p_{2\nu}\varepsilon_{\lambda'}^{\nu,*}(p_2) = 0$ already applied. Then the colour and polarization sum of the matrix element squared can be performed,

$$\sum_{\text{col.,pol.}} |\mathcal{M}^{ab\lambda\lambda'}|^2 = 8(D-2)|F_1(p_1,p_2,k)|^2|_{(p_1+p_2)^2=M_H^2}$$

and, taking only the dominant contribution from the top quarks into account, the partial decay width for a Higgs boson is

$$\Gamma(H\to gg) = \frac{\alpha_s^2}{8\pi^3 M_H}\frac{m_t^4}{v^2}\left[(4m_t^2 - M_H^2)C_0(0,0,M_H^2,m_t^2,m_t^2,m_t^2) + 2\right]^2\ .$$

In the limit of the top quark being much heavier than the Higgs boson, $m_t \gg M_H$, this expression simplifies to

$$\Gamma(H\to gg) = \frac{\alpha_s^2}{72\pi^3}\frac{M_H^3}{v^2}\ .$$

Similarly, the gluon-fusion production cross section $\sigma(gg\to H)$ can be calculated.