Exercises on Supersymmetry **Sheet 8**

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Exercise 17 *Superfield of the abelian field-strength tensor* (7 points)

The components of the vector superfield of an abelian gauge symmetry in Wess–Zumino gauge reads

$$V(x,\theta,\bar\theta)\Big|_{\mathrm{WZ}} = -(\theta\bar\sigma^\mu\bar\theta)A_\mu(x) - i(\theta\theta)\bar\theta\bar\lambda(x) + i(\bar\theta\bar\theta)\theta\lambda(x) - \tfrac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D(x). \qquad (1)$$

The corresponding (SUSY gauge-invariant) superfield of the abelian field-strength tensor is defined as

$$W_a(x,\theta,\bar\theta) = -\tfrac{1}{4}(\bar{\mathcal{D}}\bar{\mathcal{D}})\mathcal{D}_a V(x,\theta,\bar\theta). \qquad (2)$$

a) Show that $\bar W^{\dot a} \equiv \Big(W^a(x,\theta,\bar\theta)\Big)^* = +\tfrac{1}{4}(\mathcal{D}\mathcal{D})\bar{\mathcal{D}}^{\dot a}V(x,\theta,\bar\theta)$.

b) Show that $\mathcal{D}^a W_a = -\bar{\mathcal{D}}_{\dot a}\bar W^{\dot a}$.

(Hint: useful relations are found in Exercise 14.)

c) Derive the component form of W_a from its definition and from $V|_{\mathrm{WZ}}$.

(Hint: Parametrize $V|_{\mathrm{WZ}}$ first in terms of complex superspace coordinates $y^\mu = x^\mu - i\theta\bar\sigma^\mu\bar\theta$, $\eta = \theta$, and $\bar\eta = \bar\theta$. Then calculate W_a as a function of y, η, and $\bar\eta$.)

Exercise 18 *Conventional gauge transformation of the chiral superfield* (5 points)

A chiral superfield, parametrized by complex coordinates (see also previous exercise),

$$\Phi(y,\eta) = \phi(y) + \sqrt{2}\eta\psi(y) - (\eta\eta)\mathcal{F}(y) \qquad (3)$$

transforms under a conventional abelian gauge transformation according to

$$\Phi(y,\eta) \to \Phi'(y,\eta) = \exp\{-iq\omega(y)\}\,\Phi(y,\eta), \qquad (4)$$

where q denotes the abelian charge of Φ and $\omega(x)$ an arbitrary real function. Show that every component $f(x)$ of Φ, i.e. $f = \phi, \psi, \mathcal{F}$, transforms as follows:

$$f(x) \to f'(x) = \exp\{-iq\omega(x)\}\,f(x). \qquad (5)$$

Make use of Eq. (4) expressed in the original superspace coordinates x, θ, and $\bar\theta$.