Exercise 8 *Wave functions of photons* (7 points)

In Lorenz gauge the polarization vector ε^μ of a photon with momentum k^μ fulfills the condition $k \cdot \varepsilon = 0$.

a) Show that the polarization vectors $\varepsilon_\pm^\mu = \frac{1}{2}\varepsilon_{\pm,\dot{a}b}\sigma^{\mu,\dot{a}b}$ corresponding to the bispinors

$$\varepsilon_{+,\dot{a}b} = \sqrt{2}\,\frac{\bar{g}_{\dot{a}}k_b}{\langle gk\rangle^*}, \qquad \varepsilon_{-,\dot{a}b} = \sqrt{2}\,\frac{\bar{k}_{\dot{a}}g_b}{\langle gk\rangle}, \tag{1}$$

fulfill the Lorenz condition and are normalized as follows: $(\varepsilon_i^\mu)^*\varepsilon_{j,\mu} = -\delta_{ij}$. Here g is an arbitrary commuting Weyl spinor with $\langle gk\rangle \neq 0$.
(Hint: First show that $(\varepsilon_\pm^\mu)^* = \varepsilon_\mp^\mu$.)

b) Show that a redefinition of the "gauge spinor" $g \to g'$ changes the polarization vectors by an amount proportional to the momentum k.
(Hint: Calculate $\varepsilon_i - \varepsilon_i'$ with bispinors and use the Schouten identity.)

c) Consider a photon propagating in z-direction $(k^\mu) = k^0(1,0,0,1)$ and a gauge spinor $(g_a) = (0,1)$. Which helicities correspond to the polarization vectors ε_i? Does the identification depend on the direction of k^μ?

Exercise 9 *Partial proof of the Haag–Lopuszanski–Sohnius theorem* (7 points)

Let $Q_{a,r}$ and $\bar{Q}_{\dot{a},r}$, $(a = 1,2,\ r = 1,\dots,N)$ be the fermionic generators of the SUSY algebra and P^μ the momentum generator. Using the Poincaré algebra, the transformation behaviour of the fermionic generators as well as the relation

$$\left\{Q_{a,r}, \bar{Q}_{\dot{b},s}\right\} = 2\delta_{rs}\bar{\sigma}^\mu_{a\dot{b}}P_\mu, \tag{2}$$

show that

$$[P^\mu, Q_{a,r}] = 0, \qquad \left[P^\mu, \bar{Q}_{\dot{a},r}\right] = 0. \tag{3}$$

Guidance:
Justify the general ansatz

$$[P_{\dot{a}b}, Q_{c,r}] = \epsilon_{bc}X_{rs}\bar{Q}_{\dot{a},s}, \qquad \left[P_{\dot{b}a}, \bar{Q}_{\dot{c},r}\right] = -\epsilon_{\dot{b}\dot{c}}X^*_{rs}Q_{a,s} \tag{4}$$

with a numerical $N \times N$-matrix X. Using the Jacobi identity twice in

$$\left[P_{\dot{a}b}, \left[P_{\dot{c}d}, \left\{Q_{e,r}, \bar{Q}_{\dot{f},s}\right\}\right]\right], \tag{5}$$

deduce that $XX^\dagger = X^*X = XX^* = 0$. How can you conclude that $X = 0$?