Exercise 7.1 (6 points) *Gluon fields in axial gauge*

The gauge of the gluons in QCD can be fixed by the Lagrangian

$$\mathcal{L}_{\text{fix}} = -\frac{1}{2\xi}(n^\mu A^a_\mu)^2,$$

where n^μ denotes a constant 4-vector.

a) Applying this gauge, calculate the free gluon propagator in momentum space, $G^{AA,ab}_{0,\mu\nu}(k,-k)$.

b) What is the Faddeev–Popov Lagrangian $\mathcal{L}_{\text{FP}}$ corresponding to this gauge fixing? Which Feynman rule can be obtained for the coupling of the gluons to the ghost fields?

c) Derive the Slavnov–Taylor identity

$$n^\mu n^\nu G^{AA,ab}_{\mu\nu}(k,-k) = -i\xi\delta^{ab}$$

by starting from the BRS variation of $\langle 0|T\bar{u}^a(x)A^b_\nu(y)|0\rangle$. Check the identity for the free propagator.

d) The "axial gauge" is defined by the limit $\xi \to 0$. Explain why, in this gauge, no Feynman graphs with ghost fields contribute to the S matrix elements.
(Hint: The polarisation vectors $\varepsilon^a_\mu(k)$ of the external gluons fulfill the condition $n^\mu\varepsilon^a_\mu = 0$.)

Please turn over!

Exercise 7.2 (4 points) *Ward identity of the fermion–photon vertex in QED*

The vertex function $\Gamma_\mu^{A\bar{f}f}(k,\bar{p},p)$ of the fermion–photon vertex of QED is related to the Green function $G_\mu^{A\bar{f}f}(k,\bar{p},p)$ in momentum space according to

$$G_\mu^{A\bar{f}f}(k,\bar{p},p) \;=\; G_{\mu\nu}^{AA}(k,-k)\, G^{\bar{f}f}(\bar{p},-\bar{p})\, \Gamma^{A\bar{f}f,\nu}(k,\bar{p},p)\, G^{\bar{f}f}(-p,p),$$

where $G_{\mu\nu}^{AA}(k,-k)$ and $G^{\bar{f}f}(-q,q)$ denote the photon and the fermion propagators. The vertex functions can additionally be decomposed in the following way,

$$\begin{aligned}
\Gamma_\mu^{A\bar{f}f}(k,\bar{p},p) &= -iQ_f e\left[\gamma_\mu + \Lambda_\mu(-\bar{p},p)\right], \\
\Gamma^{\bar{f}f}(-q,q) &= -\left[G^{\bar{f}f}(-q,q)\right]^{-1} \;=\; i\left[\not{q} - m + \Sigma^{\bar{f}f}(q)\right],
\end{aligned}$$

where the vertex correction $\Lambda_\mu(-\bar{p},p)$ and the self energy $\Sigma^{\bar{f}f}(q)$ include quantum corrections of higher orders of perturbation theory.

a) Which relation between the vertex function $\Gamma_\mu^{A\bar{f}f}$ and $\Gamma^{\bar{f}f}$ results from the following Ward identities?

$$\begin{aligned}
\frac{i}{\xi}k^2 k^\mu\, G_\mu^{A\bar{f}f}(k,\bar{p},p) &= Q_f e\left[G^{\bar{f}f}(-p,p) - G^{\bar{f}f}(\bar{p},-\bar{p})\right], \\
k^\mu G_{\mu\nu}^{AA}(k,-k) &= -i\xi\frac{k_\nu}{k^2}.
\end{aligned}$$

b) Which relation between Λ_μ and $\Sigma^{\bar{f}f}$ can be obtained from these identities? In particular, consider the “Thomson limit” $k \to 0$.