

Exercise 11.1 (4 points) *Contribution of a spin-0 particle to the β -function of QCD*

- a) Using the result of Exercise 10.1, determine the contribution of a spin-0 particle in the fundamental representation of SU(3) to the wave-function renormalization constant δZ_A of the gluons at one-loop order in the $\overline{\text{MS}}$ scheme.
- b) Determine the coupling renormalization constant δZ_g in QCD with N_F Dirac fermions and N_S spin-0 particles at one-loop order and, with this result, derive the β -function $\beta(\alpha_s) = \mu^2 \partial \alpha_s(\mu^2) / \partial \mu^2$ with the renormalized coupling $\alpha_s(\mu^2) = Z_g^{-2} \alpha_{s,0}$.

Hint: Determine the coupling renormalization constant imposing the renormalization condition for the quark–gluon vertex $\Lambda_\mu^{g\bar{q}q}|_{\text{UV-div}} + \frac{\lambda^a}{2} \gamma_\mu \left(\delta Z_g + \delta Z_q + \frac{\delta Z_A}{2} \right) = 0$ and use $\Lambda_\mu^{g\bar{q}q}|_{\text{UV-div}} = \frac{\lambda^a}{2} \gamma_\mu \frac{\alpha_s}{4\pi} (C_F + C_A) \Delta_{\text{UV}}$.

Exercise 11.2 (6 points) *Renormalization of the Higgs–quark vertex*

In the Standard Model of particle physics the interaction of the scalar *Higgs boson* H with a quark q with mass m_q (with field variable ψ) is given by the so-called *Yukawa coupling*

$$\mathcal{L}_Y = -y_q H \bar{\psi} \psi$$

with $y_q = m_q/v$ and $v = 246$ GeV.

- a) Derive the counterterm vertex to the Yukawa interaction \mathcal{L}_Y in one-loop approximation in QCD, i.e. to order $\mathcal{O}(\alpha_s)$. In this order no wave-function renormalization of the Higgs boson is necessary, i.e. $\delta Z_H = 0$. The wave-function renormalization constant of a quark in the $\overline{\text{MS}}$ scheme is

$$\delta Z_\psi = -\frac{\alpha_s}{4\pi} C_F \Delta_{\text{UV}}, \quad \Delta_{\text{UV}} = \frac{2}{4 - D_{\text{UV}}} - \gamma_E + \ln(4\pi),$$

and the bare Yukawa coupling $y_{q,0}$ is renormalized by the renormalization constant δZ_y with $y_{q,0} = (1 + \delta Z_y) y_q$. How are the unrenormalized vertex function $\Gamma^{H\bar{q}q}(k, -p', p)$ and the renormalized vertex function $\hat{\Gamma}^{H\bar{q}q}(k, -p', p)$ related?

- b) Calculate the UV divergence in the vertex function $\Gamma^{H\bar{q}q}(k, -p', p)$ to one-loop order explicitly. First, give an argument why it is sufficient for the calculation of the UV-divergent part to consider the vertex with vanishing external momenta, $\Gamma^{H\bar{q}q}(0, 0, 0)$.
- c) Using the results from a) and b), calculate the renormalization constant δZ_y in the $\overline{\text{MS}}$ scheme.