Exercises to Relativistic Quantum Field Theory — Sheet 6
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Exercise 6.1 *Momentum of the quantised free scalar field (cont'd)* (1 point)

a) How does P^μ from Exercise 5.1 act on the one-particle wave function $\varphi_{\vec{p}}(t,\vec{x}) = \langle 0|\phi(t,\vec{x})|\vec{p}\rangle$ with fixed momentum $\vec{p}$? The action of an operator A on $\varphi_{\vec{p}}(t,\vec{x})$ is defined by $A\varphi_{\vec{p}}(t,\vec{x}) = \langle 0|\phi(t,\vec{x})A|\vec{p}\rangle$.

b) Express the one-particle wave function $\phi_f(t,\vec{x})$ corresponding to the state

$$|f\rangle = \int d\tilde{p}\; f(\vec{P})|\vec{p}\rangle$$

in terms of $\phi_{\vec{p}}(t,\vec{x})$. Show that $\phi_f(t,\vec{x})$ satisfies the Klein-Gordon equation.

Exercise 6.2 *Identities of the scalar field operator* (1 point)

Consider the field operator $\phi(x)$ of the free, real Klein-Gordon field.

a) Show that

$$[\phi(x),\phi(y)] = \int d\tilde{k}\left(\mathrm{e}^{-ik(x-y)} - \mathrm{e}^{+ik(x-y)}\right)$$

and argue why $[\phi(x),\phi(y)] = 0$ for $(x-y)^2 < 0$, as demanded by causality.

b) Prove the relation between time ordering and normal ordering:

$$\begin{aligned} T\left[\phi(x)\phi(y)\right] &\equiv \theta(x_0-y_0)\phi(x)\phi(y) + \theta(y_0-x_0)\phi(y)\phi(x) \\ &= :\phi(x)\phi(y): + \langle 0\,|T\left[\phi(x)\phi(y)\right]|\,0\rangle. \end{aligned}$$

Exercise 6.3 *Normalization of multi-particle states* (0.5 points)

Show that the n-particle states in (bosonic) Fock space,

$$|\vec{p}_1,\ldots\vec{p}_n\rangle = a^\dagger(\vec{p}_1)a^\dagger(\vec{p}_2)\ldots a^\dagger(\vec{p}_n)|0\rangle,$$

are normalised according to

$$\langle\vec{p}_1,\ldots\vec{p}_n|\vec{k}_1,\ldots,\vec{k}_m\rangle = \delta_{mn}\,(2\pi)^{3n}\sum_{\pi\in S_n}\prod_i(2p_i^0)\,\delta^3(\vec{p}_i-\vec{k}_{\pi(i)}),$$

where the sum runs over all permutations S_n of the indices $\{1,\ldots,n\}$.