
Exercises to Relativistic Quantum Field Theory — Sheet 11

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Exercise 11.1 *Pair production of scalars in the Yukawa model* (2 points)

Consider the pair production of two identical, neutral scalar particles S that are produced via fermion–antifermion annihilation

$$f(p_1) + \bar{f}(p_2) \rightarrow S(k_1) + S(k_2),$$

where the momentum assignment of the respective particles are indicated in brackets. In the centre-of-mass system the momenta are given by

$$p_{1,2}^\mu = E(1, 0, 0, \pm\beta_f), \quad k_{1,2}^\mu = E(1, \pm\beta_S \sin\theta \cos\varphi, \pm\beta_S \sin\theta \sin\varphi, \pm\beta_S \cos\theta),$$

where E is the beam energy and $\beta_f = \sqrt{1 - m_f^2/E^2}$, $\beta_S = \sqrt{1 - m_S^2/E^2}$ are the velocities of the respective particles of masses m_f and m_S . The Dirac fermion f (field ψ) and the scalar S (field ϕ) interact via a pure Yukawa interaction described by the Lagrangian

$$\mathcal{L}_I = -y\bar{\psi}\psi\phi,$$

with y denoting a (dimensionless) coupling constant.

- a) Draw all relevant Feynman diagrams for the transition matrix element \mathcal{M} in lowest perturbative order and write down the explicit expression for \mathcal{M} . How does \mathcal{M} behave under the interchange $k_1 \leftrightarrow k_2$ and why?
- b) Calculate the spin-averaged squared transition matrix element $\overline{|\mathcal{M}|^2} = \frac{1}{4} \sum_{\text{Pol.}} |\mathcal{M}|^2$ and show that

$$\overline{|\mathcal{M}|^2} = \frac{y^4}{2} \left[\frac{1}{t - m_f^2} - \frac{1}{u - m_f^2} \right]^2 \left[ut + m_f^2 s - (m_f^2 + m_S^2)^2 \right].$$

- c) Derive both the differential cross section $d\sigma/d\cos\theta$ and the total cross section σ .
- d) Draw all Feynman graphs for \mathcal{M} of order y^4 , i.e. in 1-loop approximation, which contribute to this process.

Please turn over!

Exercise 11.2 *Free photon field in radiation gauge* (1 point)

The field operator of the free photon field in radiation gauge ($A^0 = 0, \nabla \vec{A} = 0$) is given by

$$A^\mu(x) = \int d\vec{k} \sum_{\lambda=\pm} \left[e^{-ikx} \varepsilon_\lambda^\mu(k) a_\lambda(\vec{k}) + e^{+ikx} \varepsilon_\lambda^\mu(k)^* a_\lambda^\dagger(\vec{k}) \right] \Big|_{k_0=|\vec{k}|}$$

with the creation and annihilation operators $a_\lambda^\dagger(\vec{k})$ and $a_\lambda(\vec{k})$, which are normalised as in the lecture. The polarisation vectors $\varepsilon_\pm^\mu(k)$ are defined as

$$\varepsilon_\pm^\mu(\hat{k}) = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0) \quad \text{for} \quad \hat{k}^\mu = \hat{k}_0(1, 0, 0, 1)$$

and analogously for other directions.

a) Verify the polarisation sum

$$\sum_{\lambda=\pm} \varepsilon_\lambda^m(k) \varepsilon_\lambda^n(k)^* = \delta^{mn} - \frac{k^m k^n}{k^2} \quad \text{for} \quad m, n = 1, 2, 3.$$

b) The field variable that is canonical conjugate to A^m is $\Pi^m = F^{m0}$. Calculate the canonical equal-time commutators, i.e. $[A^m(t, \vec{x}), A^n(t, \vec{y})]$, $[A^m(t, \vec{x}), \Pi^n(t, \vec{y})]$, $[\Pi^m(t, \vec{x}), \Pi^n(t, \vec{y})]$. Make use of the “transverse δ -function” when appropriate,

$$\delta_{\text{tr}}^{mn}(\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3} \left(\delta^{mn} - \frac{k^m k^n}{k^2} \right) e^{-i\vec{k}\vec{x}}, \quad m, n = 1, 2, 3.$$

Exercise 11.3 *Massive gauge-boson propagator in covariant gauge* (1 point)

The propagator $D_\xi^{\mu\nu}(x)$ of a massive vector boson of mass M in covariant gauge is defined by

$$\left[g_{\mu\nu}(\square + M^2) + \left(\frac{1}{\xi} - 1 \right) \partial_\mu \partial_\nu \right] D_\xi^{\nu\rho}(x) = \delta_\mu^\rho \delta(x).$$

Calculate the Fourier transform $\tilde{D}_\xi^{\mu\nu}(q)$ of the propagator upon inserting

$$D_\xi^{\mu\nu}(x) = \int \frac{d^4q}{(2\pi)^4} \exp(-iqx) \tilde{D}_\xi^{\mu\nu}(q).$$

Here it is useful to employ the decomposition of $\tilde{D}_\xi^{\mu\nu}(q)$ into its transverse part $\tilde{D}_{T,\xi}(q)$ and its longitudinal part $\tilde{D}_{L,\xi}(q)$, so that

$$\tilde{D}_\xi^{\mu\nu}(q) = \tilde{D}_{T,\xi}(q) \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) + \tilde{D}_{L,\xi}(q) \frac{q^\mu q^\nu}{q^2}.$$

Determine the limit $\xi \rightarrow \infty$ of $\tilde{D}_\xi^{\mu\nu}(q)$.