**Exercises on Supersymmetry** **Sheet 11**

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**Exercise 22** *Sfermion sector of the MSSM* (5 points)

Assuming "minimal flavour violation" one obtains the following mass terms for sfermions $\tilde{f}$,

$$\mathcal{L}_{\mathrm{mass},\tilde{f}} = -\left(\hat{\tilde{f}}_{\mathrm{L}}^{\dagger}, \hat{\tilde{f}}_{\mathrm{R}}^{\dagger}\right) Z_{\tilde{f}} \begin{pmatrix} \hat{\tilde{f}}_{\mathrm{L}} \\ \hat{\tilde{f}}_{\mathrm{R}} \end{pmatrix}, \qquad Z_{\tilde{f}} = \begin{pmatrix} m_f^2 + M_{\tilde{f}}^{\mathrm{LL}} & m_f \left(M_{\tilde{f}}^{\mathrm{LR}}\right)^* \\ m_f M_{\tilde{f}}^{\mathrm{LR}} & m_f^2 + M_{\tilde{f}}^{\mathrm{RR}} \end{pmatrix}, \tag{1}$$

where $\hat{\tilde{f}}_{\mathrm{L/R}}$ are the sfermion fields that correspond to fermion fields $\Psi_f$ in the basis of fermion mass eigenstates. The coefficients of the mass matrix $Z_{\tilde{f}}$ are given as

$$M_{\tilde{f}}^{\mathrm{LL}} = M_{\mathrm{Z}}^2 \cos(2\beta)\left(I_{\mathrm{w},f_{\mathrm{L}}}^3 - Q_f s_{\mathrm{w}}^2\right) + M_{\tilde{F}_{\mathrm{L}}}^2, \qquad I_{\mathrm{w},f_{\mathrm{L}}}^3 = \pm\frac{1}{2}, \tag{2a}$$

$$M_{\tilde{f}}^{\mathrm{LR}} = A_f - \mu^* \begin{cases} \cot\beta, & I_{\mathrm{w},f_{\mathrm{L}}}^3 = +\frac{1}{2} \\ \tan\beta, & I_{\mathrm{w},f_{\mathrm{L}}}^3 = -\frac{1}{2} \end{cases}, \tag{2b}$$

$$M_{\tilde{f}}^{\mathrm{RR}} = M_{\mathrm{Z}}^2 \cos(2\beta) Q_f s_{\mathrm{w}}^2 + M_{\tilde{f}_{\mathrm{R}}}^2. \tag{2c}$$

Diagonalization of the mass matrix $Z_{\tilde{f}}$ yields

$$\mathcal{L}_{\mathrm{mass},\tilde{f}} = -\sum_{k=1,2} m_{\tilde{f}_k}^2 \, \tilde{f}_k^{\dagger} \tilde{f}_k, \tag{3}$$

with new sfermion fields

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = U_{\tilde{f}} \begin{pmatrix} \hat{\tilde{f}}_{\mathrm{L}} \\ \hat{\tilde{f}}_{\mathrm{R}} \end{pmatrix}, \qquad U_{\tilde{f}} = \begin{pmatrix} \cos\theta_{\tilde{f}} & \sin\theta_{\tilde{f}} \\ -\sin\theta_{\tilde{f}} & \cos\theta_{\tilde{f}} \end{pmatrix}, \tag{4}$$

where $\theta_{\tilde{f}}$ is a real mixing angle. Calculate the mass values $m_{\tilde{f}_k}^2$ (by definition $m_{\tilde{f}_1} \le m_{\tilde{f}_2}$) and show the following useful relations,

$$\cos(2\theta_{\tilde{f}}) = \frac{M_{\tilde{f}}^{\mathrm{LL}} - M_{\tilde{f}}^{\mathrm{RR}}}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2}, \qquad \sin(2\theta_{\tilde{f}}) = \frac{2m_f \left[A_f - \mu\{\cot\beta, \tan\beta\}\right]}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2}, \tag{5}$$

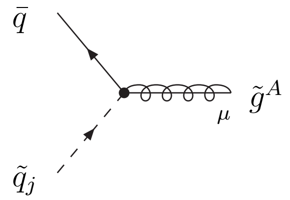
with $\{\cot\beta, \tan\beta\}$ denoting the alternatives for $I_{\mathrm{w},f_{\mathrm{L}}}^3 = \pm\frac{1}{2}$. What happens in the limiting cases of

a) small fermion mass $m_f$,

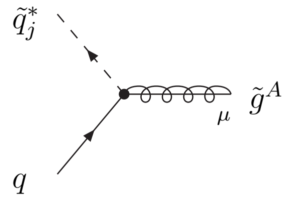
b) large SUSY parameters $M_{\tilde{F}_{\mathrm{L}}}$, $M_{\tilde{f}_{\mathrm{R}}}$, $A_f$, and $\mu$, which should scale proportional to a large scale $M_{\mathrm{SUSY}}$?

**Exercise 23** *SUSY QCD correction to the quark self-energy* (5 bonus points)

The Feynman rules for the quark–squark–gluon interaction are as follows,

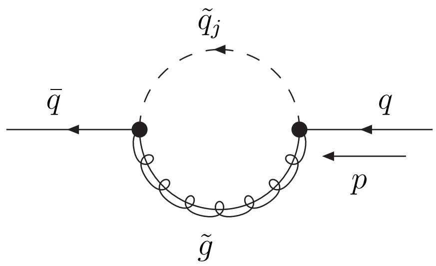



$$i\sqrt{2}g_{\mathrm{s}}\frac{\lambda^A}{2}\left(-U^*_{\tilde{q},j1}\omega_+ + U^*_{\tilde{q},j2}\omega_-\right)$$




$$i\sqrt{2}g_{\mathrm{s}}\frac{\lambda^A}{2}\left(U_{\tilde{q},j2}\omega_+ - U_{\tilde{q},j1}\omega_-\right)$$

where $U_{\tilde{q}}$ is the mixing matrix from Exercise 22 and $\omega_\pm = \frac{1}{2}(1 \pm \gamma_5)$ are the chiral projectors. Calculate the one-loop contribution to the quark self-energy $i\Sigma^{\bar{q}q}(p)$ with momentum transfer $p$, induced by exchange of a SUSY particle which is mediated by the following diagram,




Parametrize the loop integrals by the following standard functions,

$$B_0(p^2, m_0, m_1) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\,\frac{1}{(q^2 - m_0^2 + i0)\,[(q+p)^2 - m_1^2 + i0]}, \tag{6}$$

$$B_\mu(p, m_0, m_1) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\,\frac{q_\mu}{(q^2 - m_0^2 + i0)\,[(q+p)^2 - m_1^2 + i0]} \tag{7}$$

$$= p_\mu B_1(p^2, m_0, m_1), \tag{8}$$

and decompose the self-energy into its vector-, axial vector-, and scalar parts according to the relation

$$\Sigma^{\bar{q}q}(p) = \not{p}\,\Sigma^{\bar{q}q}_{\mathrm{V}}(p^2) + \not{p}\gamma_5\,\Sigma^{\bar{q}q}_{\mathrm{A}}(p^2) + m_q\mathbf{1}\,\Sigma^{\bar{q}q}_{\mathrm{S}}(p^2). \tag{9}$$

Calculate the limit of large SUSY parameters (here equivalent to $p^2, m_q \to 0$) analogously to Exercise 22.