Exercise 21 *Higgs sector of the MSSM* (13 points)

In the MSSM the Higgs sector consists of two complex SU(2) doublets that shall be parametrized as follows:

$$H^1 = \begin{pmatrix} H_1^1 \\ H_2^1 \end{pmatrix} = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + h_1 + i\chi_1) \end{pmatrix}, \qquad H^2 = \begin{pmatrix} H_1^2 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_2 + h_2 - i\chi_2) \\ -\phi_2^- \end{pmatrix}, \tag{1}$$

where h_k, χ_k denote real and ϕ_k^+, $\phi_k^- = (\phi_k^+)^\dagger$ complex fields. The constants $v_k > 0$ describe the vacuum expectation values of the Higgs fields. Each of the scalar fields H^k ($k = 1, 2$) denotes the component field with lowest (mass) dimension of a chiral superfield $\Phi_{H^k}(x, \theta, \bar{\theta})$, which again forms a SU(2) doublet.

The covariant derivative of the $\mathrm{SU}(2)_I \times \mathrm{U}(1)_Y$ gauge symmetry for SU(2) doublets is

$$D_\mu = \partial_\mu + ig\frac{\sigma^A}{2}W_\mu^A + ig'\frac{Y}{2}B_\mu, \tag{2}$$

where the hypercharges of the Higgs doublets (as well as their corresponding chiral superfields Φ_{H^k}) are $Y_{H^1} = 1$ and $Y_{H^2} = -1$. Here g and g' are the gauge couplings of the $\mathrm{SU}(2)_I$ and $\mathrm{U}(1)_Y$, and W_μ^A ($A = 1, 2, 3$) and B_μ the corresponding gauge fields. The matrices σ^A denote the usual Pauli matrices. The gauge fields W_μ^A and B_μ are component fields of their corresponding vector superfields $V_I^A(x, \theta, \bar{\theta})$ and $V_Y(x, \theta, \bar{\theta})$.

a) The kinetic part of the Higgs superfields in the Lagrangian is

$$\mathcal{L}_{\mathrm{kin},H} = -\frac{1}{2}\sum_{k=1}^{2} \Phi_{H^k}^\dagger \exp\left\{-g\sigma^A V_I^A - g'YV_Y\right\} \Phi_{H^k}\Big|_D. \tag{3}$$

In $\mathcal{L}_{\mathrm{kin},H}$, identify the mass terms of the gauge bosons W and Z,

$$\mathcal{L}_{\mathrm{mass},WZ} = M_W^2 W_\mu^+ W^{-,\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu, \tag{4}$$

where the fields are defined as

$$W_\mu^\pm = \frac{1}{2}\left(W_\mu^1 \mp iW_\mu^2\right), \qquad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}. \tag{5}$$

Here we abbreviated $s_w = \sin\theta_w$ and $c_w = \cos\theta_w$, which are defined by the relation $\tan\theta_w = g'/g$. How are the gauge-boson masses M_W and M_Z expressed in terms of the parameters v_k, g, and g'?

(Solution: $M_W = c_w M_Z = \frac{1}{2}g\sqrt{v_1^2 + v_2^2}$.)

b) After elimination of the auxiliary fields in $\mathcal{L}_{\text{MSSM}}$ the Higgs fields are subject to self-interactions given by the potential

$$\begin{aligned} V_{\text{Higgs}}(H^1, H^2) = (\mu^2 + m_1^2) H^{1\dagger} H^1 + (\mu^2 + m_2^2) H^{2\dagger} H^2 + m_{12}^2 (H^{1T} \epsilon H^2 + H^{1\dagger} \epsilon H^{2*}) \\ + \frac{1}{8}(g^2 + g'^2)(H^{1\dagger} H^1 - H^{2\dagger} H^2)^2 + \frac{1}{2} g^2 (H^{1\dagger} H^2)(H^{2\dagger} H^1), \quad (6) \end{aligned}$$

where terms proportional to μ^2 come from the superpotential, those proportional to m_k^2 ($k = 1, 2, 12$) parametrize a part of the soft SUSY breaking. The parameters μ and m_k are assumed to be real.

Which (two) equations encode the requirement that V_{Higgs} possesses no terms linear in the fields h_k, χ_k, and $\phi_k^\pm$ (ground-state requirement)? Is it possible to satisfy these conditions with $v_k \neq 0$ if the SUSY breaking terms m_k vanish?

c) Calculate the terms in V_{Higgs} that are bilinear in the fields h_k, χ_k, and $\phi_k^\pm$. These terms define $V_{\text{Higgs,quad}}$. Using the conditions from b) one eliminates m_1 and m_2.

Our aim is to extract the fields corresponding to mass eigenstates from $V_{\text{Higgs,quad}}$ and the corresponding mass eigenvalues. To this end, first perform the following orthogonal transformation,

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}, \qquad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}, \quad (7)$$

to identify the mass terms for the scalar fields G^0, $G^\pm$, A^0, and $H^\pm$,

$$V_{\text{Higgs,mass}}\Big|_{G^0, G^\pm, A^0, H^\pm} = \frac{1}{2} M_A^2 (A^0)^2 + M_{H^\pm}^2 H^+ H^-. \quad (8)$$

How is $\tan\beta$ expressed as a function of v_k? What are the results for M_A and $M_{H^\pm}$?

d) The values M_h and M_H for the masses of the remaining two Higgs bosons h and H, where

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad (9)$$

result from diagonalization of the matrix $\mathcal{M}_h^2$ that connects h_1 and h_2 in $V_{\text{Higgs,mass}}$,

$$V_{\text{Higgs,mass}}\Big|_{h_1, h_2} = \frac{1}{2}(h_1, h_2) \mathcal{M}_h^2 \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \frac{1}{2} M_h^2 h^2 + \frac{1}{2} M_H^2 H^2. \quad (10)$$

Calculate M_h and M_H and express the results in terms of the parameters M_A, M_Z, and $\cos(2\beta)$. What is the upper bound for the mass M_h of the (by definition) lightest Higgs boson h if you take M_Z as a fixed input parameter and vary M_A and $\cos(2\beta)$?