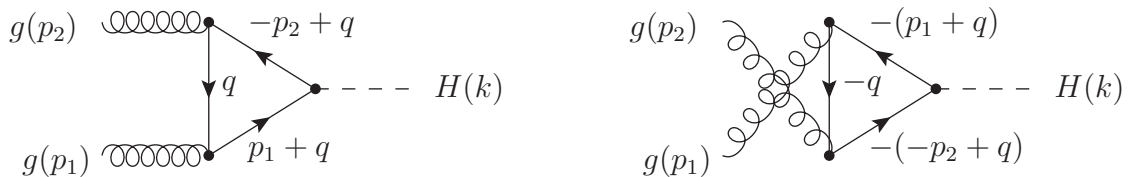


Exercise 12.1 (10 points) *Higgs–gluon vertex*

Via the Yukawa interaction discussed in Exercise 11.2, quark loops induce an effective Higgs–gluon coupling which, at one-loop level, is mediated by the following diagrams:



This process is the dominant production channel for the Higgs boson at the Tevatron and the LHC.

- a) Show that, for on-shell gluons ($p_1^2 = p_2^2 = 0$), the vertex function of the Higgs–gluon vertex can be decomposed into the form

$$\Gamma_{\mu\nu}^{g_a g_b H}(p_1, p_2, k) = i\delta^{ab} \left[F_1(p_1, p_2, k) \left(g_{\mu\nu} - \frac{p_{2\mu} p_{1\nu}}{(p_1 \cdot p_2)} \right) + F_2(p_1, p_2, k) \frac{p_{1\mu} p_{2\nu}}{(p_1 \cdot p_2)} \right].$$

Start with the general ansatz for $\Gamma_{\mu\nu}^{g_a g_b H}$ in form of the linear combination of all tensors of rank 2 which can be built with $g_{\mu\nu}$ and the momenta $p_{1,2}$. Apply then the gauge-invariance conditions $p_1^\mu \Gamma_{\mu\nu}^{ggH} = p_2^\nu \Gamma_{\mu\nu}^{ggH} = 0$.

- b) Calculate the form factor $F_1(p_1, p_2, k)$ for $(p_1 + p_2)^2 = M_H^2$, where M_H is the mass of the Higgs boson. Proceed with the following steps:

- (1) By parameterizing the loop momentum as given in the diagrams, the numerators of both diagrams can be combined. Derive the following form for the sum of both diagrams:

$$\Gamma_{\mu\nu}^{g_a g_b H}(p_1, p_2, k) = -\mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{c_{ggH} \delta_{ab} \text{Tr}[N_{\mu\nu}(p_1, p_2, k, m_q)]}{[q^2 - m_q^2][(p_1 + q)^2 - m_q^2][(-p_2 + q)^2 - m_q^2]}.$$

The factor c_{ggH} comprises all coupling constants, colour factors, and powers of i , and m_q is the mass of the quark in the loop.

Please turn over!

- (2) Project the tensor structure of $\Gamma_{\mu\nu}^{g_a g_b H}$ on the form factor F_1 with the help of the identity

$$P_{\mu\nu} P^{\mu\nu} = (D - 2), \quad P^{\mu\nu} \equiv g^{\mu\nu} - \frac{p_2^\mu p_1^\nu}{(p_1 \cdot p_2)}.$$

The trace in the numerator results in

$$\text{Tr}[N^{\mu\nu} P_{\mu\nu}] = 4m_q \left[2(D - 1)m_q^2 - (D - 2)M_H^2 - 2(D - 5)q^2 - \frac{16(p_1 \cdot q)(p_2 \cdot q)}{M_H^2} \right].$$

- (3) Reduce the loop integral to scalar integrals. The limit of $D \rightarrow 4$ results in

$$F_1(p_1, p_2, k) = -\frac{\alpha_s}{2\pi} \frac{m_q^2}{v} \left[(4m_q^2 - M_H^2)C_0(0, 0, M_H^2, m_q^2, m_q^2, m_q^2) + 2 \right].$$

Outlook: The results above can be used to calculate the partial decay width of a Higgs boson into gluons. The matrix element $\mathcal{M}^{ab\lambda\lambda'}$ can be expressed using the form factor F_1 as

$$\mathcal{M}^{ab\lambda\lambda'} = i\delta^{ab} F_1(p_1, p_2, k) \left(g_{\mu\nu} - \frac{p_{2\mu} p_{1\nu}}{(p_1 \cdot p_2)} \right) \varepsilon_\lambda^{\mu,*}(p_1) \varepsilon_{\lambda'}^{\nu,*}(p_2),$$

with $\varepsilon_\lambda, \varepsilon_{\lambda'}$ being the polarization vectors of the gluons and the on-shell conditions $p_{1\mu} \varepsilon_\lambda^{\mu,*}(p_1) = p_{2\nu} \varepsilon_{\lambda'}^{\nu,*}(p_2) = 0$ already applied. Then the colour and polarization sum of the matrix element squared can be performed,

$$\sum_{\text{col.,pol.}} |\mathcal{M}^{ab\lambda\lambda'}|^2 = 8(D - 2) |F_1(p_1, p_2, k)|^2 \Big|_{(p_1+p_2)^2=M_H^2}$$

and, taking only the dominant contribution from the top quarks into account, the partial decay width for a Higgs boson is

$$\Gamma(H \rightarrow gg) = \frac{\alpha_s^2}{8\pi^3 M_H} \frac{m_t^4}{v^2} \left[(4m_t^2 - M_H^2)C_0(0, 0, M_H^2, m_t^2, m_t^2, m_t^2) + 2 \right]^2.$$

In the limit of the top quark being much heavier than the Higgs boson, $m_t \gg M_H$, this expression simplifies to

$$\Gamma(H \rightarrow gg) = \frac{\alpha_s^2}{72\pi^3} \frac{M_H^3}{v^2}.$$

Similarly, the gluon-fusion production cross section $\sigma(gg \rightarrow H)$ can be calculated.