Exercise 9.1 (1.5 points) *Vertex functions of scalar QED*

The electromagnetic interaction of spin-0 particles S^{\pm} with mass M (field variable ϕ for particles S^{-} with charge Q) is described by $\mathcal{L}_{\phi} = (D_{\mu}\phi)^{\dagger}(D^{\mu}\phi) - M^{2}\phi^{\dagger}\phi$ with $D_{\mu} = \partial_{\mu} + iQeA_{\mu}$. This yields the Feynman rules of *scalar QED* given on the back side.

a) Calculate the S-boson self-energy $\Sigma^{SS}(p^2)$ in one-loop approximation in Feynman gauge. The self-energy is defined by the two-point vertex function as follows:

$$\Gamma^{S^+S^-}(-p,p) = i \left[p^2 - M^2 + \Sigma^{SS}(p^2) \right]$$

b) The three-point vertex function

$$\Gamma^{AS^+S^-}_{\mu}(k, p_+, p_-) = -iQe\Big[(p_- - p_+)_{\mu} + \Lambda_{\mu}(-p_+, p_-)\Big]$$

fulfills the Ward identity

$$k^{\mu}\Gamma^{AS^{+}S^{-}}_{\mu}(k,p_{+},p_{-}) = Qe\left[\Gamma^{S^{+}S^{-}}(-p_{-},p_{-}) - \Gamma^{S^{+}S^{-}}(p_{+},-p_{+})\right]$$

Using this identity and the fact that the UV-divergent part of $\Gamma_{\mu}^{AS^+S^-}$ is proportional to the lowest order $\Gamma_{0,\mu}^{AS^+S^-}$, derive the UV-divergent one-loop contribution to the correction $\Lambda_{\mu}(-p_+, p_-)$.

c) Analogous to QED with Dirac fermions, calculate the photon self-energy $\Sigma_{\mu\nu}^{AA}(k)$ in one-loop approximation.

Exercise 9.2 (1.5 points) Renormalization of scalar QED

a) Apply the renormalization transformation

$$e \to e(1 + \delta Z_e), \quad M^2 \to M^2 + \delta M^2, \quad A^\mu \to A^\mu \left(1 + \frac{1}{2}\delta Z_A\right), \quad \phi \to \phi \left(1 + \frac{1}{2}\delta Z_\phi\right)$$

in one-loop approximation to the Lagrangian \mathcal{L}_{ϕ} given in Exercise 9.1 and determine the generated counterterms (solution on the back side). Which counterterm contributions result for the renormalized vertex functions $\hat{\Gamma}^{S^+S^-}(-p,p)$, $\hat{\Gamma}^{AS^+S^-}_{\mu\nu}(k,p_+,p_-)$ and $\hat{\Gamma}^{AAS^+S^-}_{\mu\nu}(k_1,k_2,p_+,p_-)$?

b) Determine δM^2 from the mass-renormalization condition

$$\hat{\Gamma}^{S^+S^-}(-p,p)\Big|_{p^2=M^2} = 0.$$

c) Determine δZ_{ϕ} from the renormalization condition

$$\operatorname{Res}_{p^2=M^2}\left\{\hat{G}^{S^+S^-}(-p,p)\right\} = i$$

for the residue of the S propagator.

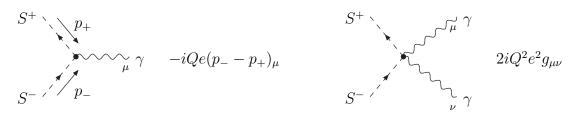
d) Determine δZ_e from the charge-renormalization condition (Thomson limit)

$$\hat{\Gamma}^{AS^+S^-}_{\mu}(k=0,-p,p)\Big|_{p^2=M^2} = -2iQep_{\mu}.$$

Using the Ward identity given in Exercise 9.1 b) show that $\delta Z_e = -\frac{1}{2} \delta Z_A$.

Feynman rules for charged spin-0 particles S^{\pm} :

Vertices:

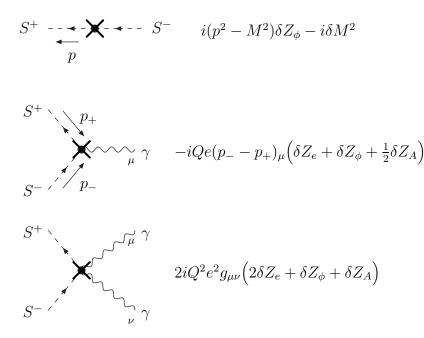


Propagator and external lines:

$$S^{+} \bullet - \bullet S^{-} \qquad \frac{i}{p^{2} - M^{2}} \qquad \bullet - \bullet - S^{+} \qquad 1$$

$$\bullet - \bullet - S^{-} \qquad 1$$

Counterterms:



The fields S^{\pm} are always assumed to be incoming at the vertex. Outgoing fields S^{\pm} correspond to the incoming fields S^{\mp} with reversed momenta. The Feynman rules for external and internal photons are the same as in QED with Dirac fermions.