Exercise 11.1 (1 point) *Contribution of a spin-0 particle to the β-function of QCD*

a) Using the result of Exercise 10.1, determine the contribution of a spin-0 particle in the fundamental representation of $SU(3)$ to the wave-function renormalization constant δZ_A of the gluons at one-loop order in the $\overline{\mathrm{MS}}$ scheme.

b) Determine the coupling renormalization constant δZ_g in QCD with N_F Dirac fermions and N_S spin-0 particles at one-loop order and, with this result, derive the β-function $\beta(\alpha_s) = \mu^2 \partial\alpha_s(\mu^2)/\partial\mu^2$ with the renormalized coupling $\alpha_s(\mu^2) = Z_g^{-2}\alpha_{s,0}$.

Hint: Determine the coupling renormalization constant imposing the renormalization condition for the quark–gluon vertex $\Lambda_\mu^{g\bar{q}q}|_{\mathrm{UV-div}} + \frac{\lambda^a}{2}\gamma_\mu\left(\delta Z_g + \delta Z_q + \frac{\delta Z_A}{2}\right) = 0$ and use $\Lambda_\mu^{g\bar{q}q}|_{\mathrm{UV-div}} = \frac{\lambda^a}{2}\gamma_\mu \frac{\alpha_s}{4\pi}(C_F + C_A)\Delta_{\mathrm{UV}}$.

Exercise 11.2 (1.5 points) *Renormalization of the Higgs–quark vertex*

In the Standard Model of particle physics the interaction of the scalar *Higgs boson H* with a quark q with mass m_q (with field variable ψ) is given by the so-called *Yukawa coupling*

$$\mathcal{L}_Y = -y_q H\,\bar{\psi}\psi$$

with $y_q = m_q/v$ and $v = 246$ GeV.

a) Derive the counterterm vertex to the Yukawa interaction $\mathcal{L}_Y$ in one-loop approximation in QCD, i.e. to order $\mathcal{O}(\alpha_s)$. In this order no wave-function renormalization of the Higgs boson is necessary, i.e. $\delta Z_H = 0$. The wave-function renormalization constant of a quark in the $\overline{\mathrm{MS}}$ scheme is

$$\delta Z_\psi = -\frac{\alpha_s}{4\pi}\,C_F\,\Delta_{\mathrm{UV}}\ , \quad \Delta_{\mathrm{UV}} = \frac{2}{4-D_{\mathrm{UV}}} - \gamma_E + \ln(4\pi),$$

and the bare Yukawa coupling $y_{q,0}$ is renormalized by the renormalization constant δZ_y with $y_{q,0} = (1+\delta Z_y)y_q$. How are the unrenormalized vertex function $\Gamma^{H\bar{q}q}(k,-p',p)$ and the renormalized vertex function $\hat{\Gamma}^{H\bar{q}q}(k,-p',p)$ related?

b) Calculate the UV divergence in the vertex function $\Gamma^{H\bar{q}q}(k,-p',p)$ to one-loop order explicitly. First, give an argument why it is sufficient for the calculation of the UV-divergent part to consider the vertex with vanishing external momenta, $\Gamma^{H\bar{q}q}(0,0,0)$.

c) Using the results from a) and b), calculate the renormalization constant δZ_y in the $\overline{\mathrm{MS}}$ scheme.