Exercise 10.1 (1.5 points) *Vertex functions of scalar QCD*

In this exercise the interaction of spin-0 particles S (field variable Φ) in the fundamental representation of SU(3) with gluons will be considered. The scalars are described by the Lagrangian $\mathcal{L}_\phi = (D_\mu\Phi)^\dagger(D^\mu\Phi) - M^2\Phi^\dagger\Phi$ with the covariant derivative $D_\mu = \partial_\mu + ig_s T^a A^a_\mu$, where $T^a = \lambda^a/2$ are the generators of SU(3).

a) Write down the Feynman rules of the scalar particle S.

b) Calculate the self-energy $\Sigma^{SS}(p^2)$ of the S bosons in one-loop approximation in the Feynman gauge. The self-energy is defined via the two-point vertex function as

$$\Gamma^{SS}(-p,p) \;=\; i\left[p^2 - M^2 + \Sigma^{SS}(p^2)\right].$$

c) Calculate the contribution of the scalars to the gluon self-energy $\Sigma^{g_a g_b}_{\mu\nu}(k)$ in one-loop approximation.

Exercise 10.2 (1.5 points) *Ghost renormalization in QCD*

The renormalized two-point vertex function of the ghost fields in QCD is given by

$$\Gamma^{\bar{u}_a u_b}(p,-p) = i\delta^{ab}\left[p^2 + \Sigma^{\bar{u}u}(p^2) + p^2\delta Z_u\right]$$

with the self-energy $\Sigma^{\bar{u}u}$ and the wave-function renormalization constant δZ_u.

a) Calculate the self-energy $\Sigma^{\bar{u}u}(p^2)$ of the ghosts in one-loop approximation in the Feynman gauge.

b) Determine δZ_u in $\overline{\mathrm{MS}}$ renormalization, so that $\Gamma^{\bar{u}_a u_b}$ is UV-finite and the renormalization constant is proportional to the constant $\Delta = \frac{2}{4-D} - \gamma_E + \ln(4\pi)$.